Lorentz Invariance Violation and the QED formation length

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Abstract

It was recently suggested that possible small violations of Lorentz invariance could explain the existence of UHECR beyond the GZK cutoff and the observations of multi-TeV gamma-rays from Mkn 501. Our analysis of Lorentz-violating kinematics shows that in addition to the modified threshold conditions solving cosmic ray puzzles we should expect a strong suppression of electromagnetic processes like bremsstrahlung and pair creation. This leads to drastic effects in electron-photon cascade development in the atmosphere and in detectors.

1 Introduction

A tiny Lorentz invariance violation (LIV) at high energies was suggested [1] as an explanation of two experimental astrophysical paradoxes - the observations of ultra high energy cosmic rays (UHECR) well beyond the theoretically expected Greisen-Zatzepin-Kuzmin (GZK) cutoff around 6×10^{19} eV [2] and the observations of 20 TeV gamma-rays from Mkn 501. UHECR should have been absorbed in photoproduction collisions with the microwave background and the 20 TeV γ–rays – on the extragalactic infrared/optical background [3]. While some authors [4,5] consider the second puzzle not so dramatic and LIV hypothesis too premature, the number of showers above 10^{20} eV is already big enough to suggest the existence of a problem. As a solution of this problem LIV was first suggested about 30 years ago [6,7] and later in [8]. In [1], [9], [10], [11] the LIV hypothesis was suggested as a solution for both paradoxes.

In Ref. [9] the Lorentz invariance violation was formulated to correspond to an energy dependent photon group velocity

$$\frac{\partial E}{\partial k} = c[1 - \xi_\gamma \frac{E}{E_0}] \ .$$

Here c is the speed of light. This corresponds to a dispersion relation

$$c^2 k^2 \simeq E^2 + \xi_\gamma \frac{E^3}{E_0} \tag{1}$$

with $\xi_\gamma = \pm 1$ and $E_0 \sim 10^{19}$ GeV.

Kifune [10] has used this formulation to investigate the consequences of LIV on collisions of high energy radiation with soft photons. The detection of TeV photons from point sources and protons above the GZK cutoff sets some constraints on ξ_γ, ξ_e and ξ_p, when photons, electrons and protons are allowed to have different degrees of the LIV. He also noted that the modified relation of energy and momentum can affect mildly the detection of high energy radiation. Observations of γ-rays are based on pair creation in the detector material (satellite experiments) or on the detection of Cherenkov light from air showers initiated by TeV photons in ground-based observations. There are also numerous experiments using electromagnetic cascading in emulsion chambers to detect high energy electrons and γ-rays.

The common wisdom is [9] that although LIV affects significantly the GZK and TeV-γ thresholds the effect of the modified dispersion relation on other interactions of the relevant high-energy particles is negligible.

The aim of the present paper is to discuss the possibility that LIV deformed dispersion relations could change not only the thresholds of some reactions at extremely high energies, but will also strongly affect the electromagnetic cascade development in the atmosphere and detectors. The reason for this is that deformed dispersion relations affect the *formation length* of bremsstrahlung and pair production and, hence, their cross sections.

2 Formation length and LI violation

The longitudinal momentum transfer between highly relativistic interacting particles (photons or electrons) and the target nuclei is small. Ter-Mikaelian [12] first realized that according to the uncertainty principle the interactions take place not at a single point but over a long distance (*formation length, coherence length*). The interactions cannot be localized within this length. In the classical electrodynamics *coherent length* is the distance over which constructive interference between radiated waves takes place. During the time the electron travels the formation length it and the radiated electromagnetic wave separate enough [14] to be considered independent particles. In bremsstrahlung this separation is at least a distance of the order of the emitted photon wavelength λ. In the case of pair production the formation length is the length

over which the electron and the positron separate by a distance of about two electron Compton wavelengths $2/m$.

While the transverse momentum exchanged with the nucleus, $q_\perp$, is of order mc, the longitudinal momentum transfer $q_{\shortparallel}$ is small. In bremsstrahlung

$$q_{\shortparallel} = p_e - p'_e - k/c \ , \tag{2}$$

where p_e and p'_e are the electron momenta before and after the interaction and k is the photon energy. At high energy $E \gg m_e c^2$ we can neglect emission angles and simplify to $q_{\shortparallel} \sim \frac{m^2 c^3 k}{2E(E-k)}$. In pair production $q_{\shortparallel} \sim \frac{m^2 c^3 k}{2E(k-E)}$. For bremsstrahlung the formation length thus is:

$$l_0 \sim \frac{\hbar}{q_{\shortparallel}} = \frac{2\hbar E(E-k)}{m^2 c^3 k} \ . \tag{3}$$

The formation length increases rapidly with the energy of the primary particle and with its ratio to the energy of the emitted photon. For pair production $E - k$ in the numerator is replaced by $k - E$.

The formation length is closely related to the interaction cross section. The amplitude of the radiation is proportional to l_0, and its intensity is proportional to $\sim l_0^2$. If there is emission from an electron traversing a distance D, this is equivalent to $\frac{D}{l_0}$ independent emitters, giving a total radiation intensity proportional to $\mid l_0 \mid^2 D/l_0 \sim l_0.$ [13].

Because of the small value of $q_{\shortparallel}$ the formation length l_0 can have macroscopic dimensions even at moderate energies. For example, for a 25 GeV electron, emitting a 100 MeV photon, $q_{\shortparallel} \sim 0.03$ eV/c and $l_0 \simeq 10 \ \mu m$. For 10^9 GeV electron and 10^5 GeV bremsstrahlung photon $q_{\shortparallel} \sim 1.45 \times 10^{-8}$ eV/c and $l_0 \sim 14$ m.

The coherence over this very long length can be disrupted by other interactions reducing the effective formation length and hence the probability for radiation. For instance, because at high energies l_0 becomes much longer than the average distance between the atoms of the medium, the multiple scattering of the electron on the atoms within the formation length will change the electron path by reducing the electron longitudinal velocity and the emission will be suppressed. This is the physical mechanism of the Landau-Pomeranchuk-Migdal (LPM) effect (see [13] and references therein). Other factors that could reduce l_0 , and hence the probability for radiation or pair production are [13] photon interaction with the medium (dielectric suppression), external magnetic field (magnetic suppression), suppression of bremsstrahlung by pair production, and vice-versa.

In the case of bremsstrahlung the multiple scattering on the formation length (LPM effect) leads to an additional term in the expression for $q_{\shortparallel}$ which in-

creases the longitudinal momentum transfer. In the small angle approximation

$$q_{\shortparallel} \simeq \frac{m^2 c^3 k}{2E(E-k)} + \frac{k\theta_{MS}^2}{2c} \ , \tag{4}$$

where θ_{MS} is the electron multiple scattering angle in half the formation length [13]. The increase of $q_{\shortparallel}$ reduces the formation length. The multiple scattering becomes significant when the second term in (4) is larger than the first.

LIV modified dispersion relation acts like the suppression factors above. Let us put, similarly to Eq. 1, the square of the modified momentum

$$q^2 = p^2 + \xi \frac{E^3}{E_0 c^2} \tag{5}$$

Then Eq. 2 becomes ($q \simeq p + \xi \frac{E^2}{2E_0 c}$)

$$q_{\shortparallel} \simeq \frac{m^2 c^3 k}{2E(E-k)} + \xi_e \frac{E^2}{2E_0 c} - \xi_e \frac{(E-k)^2}{2E_0 c} - \xi_\gamma \frac{k^2}{2E_0 c} \tag{6}$$

for bremsstrahlung, and

$$q_{\shortparallel} \simeq \frac{m^2 c^3 k}{2E(k-E)} + \xi_\gamma \frac{k^2}{2E_0 c} \tag{7}$$

for pair production. In Eq. 7 LIV dispersion relation is used only for the particle with the highest energy - the photon.

The effect depends on the signs of the parameters ξ. If we take the electron and positron energies in Eq. 7 equal to $\frac{k}{2}$ (k is the photon energy), then

$$q_{\shortparallel}^{\mathrm{min}} \simeq \frac{2m^2 c^3}{k} + \xi_\gamma \frac{k^2}{2E_0 c}$$

When $\xi_\gamma < 0$ $q_{\shortparallel}^{\mathrm{min}}$ becomes negative at the critical energy $k_{cr} = [\frac{4(mc^2)^2 E_0}{\xi_\gamma}]^{\frac{1}{3}} \simeq$ 2.2x10^{13} eV for $\mid \xi_\gamma \mid = 1$.

Let us now calculate the *suppression factor S*, that measures the relative change of the interaction cross section and is defined as:

$$S = \frac{\frac{d\sigma}{dE}}{\frac{d\sigma}{dE_{BH}}} = \frac{l_f}{l_0} \tag{8}$$

where $d\sigma/dE$ is the differential cross section with suppression, $d\sigma/dE_{BH}$ is the Bethe-Heitler (BH) cross section, and l_f is the formation length with suppression. This definition is convenient for estimation of suppression due to different factors because it is easy to estimate the change of the quantity $q_{\shortparallel}$

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Table 1
Suppression factors for pair production

$k(TeV)$	S_{LIV}	S_{LPM}^{air}	S_{LPM}^{Pb}
1	1	1	1
10	0.91	1	1
100	10^{-2}	1	0.41
1000	10^{-5}	1	0.13

caused by these factors and, respectively l_f. For example, for strong suppression by multiple scattering, $S_{LPM} = \sqrt{\frac{E_{LPM}k}{E(k-E)}}$ (the material dependent energy E_{LPM} is defined in [15]). For $E \approx k - E \approx k/2$ $S_{LPM} \simeq 2\sqrt{\frac{E_{LPM}}{k}}$.

In case of pair production LIV gives a suppression factor

$$S_{LIV} = \frac{1}{1 + \xi_\gamma \frac{k^3 u(1-u)}{E_0(mc^2)^2}} \qquad (9)$$

(here $u = E/k$) and for $E \approx k - E \approx k/2$

$$S_{LIV} \simeq \frac{1}{1 + \xi_\gamma \frac{k^3}{4E_0(mc^2)^2}} \qquad (10)$$

Neglecting 1 in the denominator for strong suppression, Eq.10 becomes

$$S_{LIV} \simeq \frac{4E_0(mc^2)^2}{\xi_\gamma k^3} \qquad (11)$$

Some numerical values for S_{LIV} for pair production ($E_0 = 10^{28}$ eV , $\xi_\gamma = 1$) are shown in table 1. For comparison LPM suppression factors for air (E_{LPM}=2.34×10^8 GeV at sea level) and lead (E_{LPM}=4.3×10^3 GeV) are also shown in the table (the values for E_{LPM} are taken from [13]).

If $\xi_\gamma = -1$

$$S_{LIV} = \frac{1}{1 - \frac{k^3}{4E_0(mc^2)^2}}$$

S_{LIV} turns into enhancement, which becomes infinite for $k = k_{thr} \equiv [4E_0(mc^2)^2]^{\frac{1}{3}} \approx 2.2 \times 10^{13}$ eV≡ 22 TeV, as shown in table 2.

This means that when ξ is negative the formation length of the process (pair production in this case), respectively the cross section, increases. This sharp

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Table 2
Enhancement factor for pair production

$k(TeV)$	S_{LIV}
1	1
10	1.11
15	1.2
20	4.27

Table 3
Suppression factors for bremsstrahlung

$E(TeV)$	S_{LIV}	S_{LPM}^{air}	S_{LPM}^{Pb}
1	1	1	6.6×10^{-2}
10	0.57	1	2.1×10^{-2}
100	1.3×10^{-3}	1	6.6×10^{-3}
1000	1.3×10^{-6}	0.48	2.1×10^{-3}

increase around k_{thr} could in some cases be compensated by multiple scattering
and other suppression factors.

The effects of LIV on bremsstrahlung are similar to those for pair production.
If we neglect the last term in (6) (we suppose that LIV is negligible for low
energy bremsstrahlung photons) the suppression factor becomes

$$S_{LIV} = \frac{1}{1 + \xi_e \frac{2E^3(1-u)}{E_0(mc^2)^2}} \tag{12}$$

or, for strong suppression,

$$S_{LIV} \simeq \frac{E_0(mc^2)^2}{2\xi_e E^3(1-u)} \tag{13}$$

The corresponding LPM suppression factor (also for strong suppression) is

$$S_{LPM} = \sqrt{\frac{E_{LPM}}{E} \frac{u}{1-u}} \tag{14}$$

Some numerical values for $\xi = 1$ and $u = 0.001$ are compared in table 3.

3 Discussion and conclusions

Using the concept of formation length we have shown that LIV parameters that are necessary for the explanation of the existence of UHECR and the non-absorption of 20 TeV γ-rays in a particular Lorentz invariance violation model will also strongly affect the bremsstrahlung and pair production cross sections. The general form of Planck scale motivated LIV dispersion relations is of the form

$$E^2 - p^2 - m^2 \simeq \eta E^2 (\frac{E}{E_0})^\alpha \simeq \eta p^2 (\frac{E}{E_0})^\alpha \ ,$$

where α and η are free parameters. We have only analyzed the case $\alpha = 1$ and $\mid \eta \mid = 1$.

To explain the experimental astrophysical paradoxes the positive values of η should be excluded [9], which means that the parameter ξ should be positive and the LIV would suppress pair production and bremsstrahlung above some critical energy k_{cr}. We obtain k_{cr} values that are of the same order of magnitude as the critical energy E_c defined in [10] (see also [16]). In the case of LIV E_c is obtained from the condition for a minimum target photon energy for pair creation in a soft photon field. Above E_c the target photon energy grows (for $\xi > 0$) until the Universe becomes transparent to ultrahigh energy photons . This reflects the fact that ([11,17]) one can expect deviations from standard kinematics when the last two terms in the dispersion relation $E^2 \approx p^2 + m^2 + \xi p^3 / E_0$ are of comparable magnitude. For $\xi = 1$ the condition becomes $p_{dev} \sim (m^2 E_0)^{\frac{1}{3}} \sim 10$ TeV.

The suppression factor S calculated above and shown for pair production in Table 1 is very strong and does not depend on the target material as in LPM. The suppression increases very fast with energy, proportionally to E^3 above 100 TeV. This will make photons and electrons very penetrating particles and will drastically suppress the electromagnetic shower development. For example, only about 20% of 100 TeV primary photons will interact in the atmosphere. For $\gtrsim 300 \div 400$ TeV photons the atmosphere will be transparent. Such an effect must have already been observed in the numerous experiments in cosmic rays. The depth of maximum in electromagnetic showers X_{max} is proportional to the product of the radiation length X_0 and the logarithm of the primary energy $\ln E$. In the case of LIV the radiation length X_0 above the critical energy becomes infinite, which changes drastically the behavior of electromagnetic and of hadronic air showers. This is especially true for giant air showers where the depth of maximum X_{max} is generally determined by the electromagnetic cascades of primary energy exceeding 10^{17} eV. Thus LIV with the parameters discussed above would contradict to the results from giant air showers [18].

Coleman and Glashow [1] have suggested a different scheme for LIV in which the maximum attainable velocity c_a of a particle is different from the photon velocity c. The relevant dispersion relations then has the form $E_a^2 = p^2 c_a^2 + m_a^2 c_a^4$. In this case our results will be applicable by substituting $c^2 - c_a^2$ for $\xi \frac{E}{E_0}$ in (1). Then the critical energy will be defined as $k_{cr} = \frac{\sqrt{8} m_e}{|c^2 - c_e^2|^{1/2}}$. If the shower development is observed with no deviation from the standard cascade theory up to $\simeq 22$ TeV, this will put the limits $E_0 \gtrsim 10^{28}$ eV (Planck mass) or $| c^2 - c_e^2 | \lesssim 1.5 \times 10^{-15}$.

The observations of giant atmospheric showers created by particles with energies $\gtrsim 10^{20}$ eV give, in principle, the possibility for a more precise test of LIV. For example, if the case $\alpha = 1$ must be ruled out, one can move to the $\alpha = 2$ case, i.e. quadratic suppression of E_0. The suppression factor for pair creation then becomes $S_{LIV} \simeq \frac{4 E_0^2 (mc^2)^2}{\xi k^4}$ and the drastic deformation of the shower development would be observed at energies $\gtrsim 10^{17}$ eV. In the frame of Coleman and Glashow scheme this will put the limits $| c^2 - c_e^2 | \lesssim 10^{-23}$. In this connection we would like to point out the interesting work [19] where new constraints on $| c_\gamma - c_{\pi^0} |$ (c_{π^0} is the maximal attainable speed of neutral pions) are obtained by comparing the experimentally measured position of the shower maximum $X_{\max}$ with calculations.

Our estimates are based on a quite general quantum-gravity induced modification of the dispersion relation between the energy and the momentum of a particle. We also supposed that the sign of the free parameter ξ is fixed. As noted above, the positive sign of ξ is required to solve the astrophysical paradoxes and is consistent with the constraints deduced from the analysis in [17]. The future studies using the fruitful and physically very clear concept of the formation (coherent) length could include some additional phenomenological suggestions made in the literature. For example, it is possible to construct schemes in which the classical relation $E^2 = p^2 + m^2$ holds only on average, but in a given physical realization $E^2 = p^2 + m^2 + \Delta$, with $-\xi p^2 (\frac{E}{E_0})^\alpha < \Delta < \xi p^2 (\frac{E}{E_0})^\alpha$ [20]. It is also possible that energy conservation (assumed in this work) may be valid only in a statistical sense (see the review [21] and references therein). If the LIV reflects some kind of special property of the space-time at Planck scales (or some other scale) the sign of ξ may fluctuate on the length scales of the order of the Planck length. The negative sign of ξ will, however, complicate the analysis. We showed above that for negative ξ the formation length increases with energy, becoming infinite at the critical energy and negative above it. It is not obvious how to interpret the infinite, or negative, formation length in interactions with the field of a nucleus.

Finally, if the LIV is obtained as a result of spontaneous symmetry breaking in the context of an explicitly Lorentz invariant theory (see, e.g. [22]), it is possible that another scale (light mass scale m_l) enters the calculations. The correction term for the modified dispersion relation in this case is of the form

(see (1)) $\xi \frac{E^2 V}{M_P c^2}$, where V is vev after the symmetry breaking and M_P might be the Planck mass. In this case we could expect much smaller LIV depending on the value of m_l. For example, if V is of the order of the other fundamental energy scale in nature, the electroweak scale $m_{EW} \sim 10^3$ GeV, then one can expect the deviation from the standard kinematics above $p_{dev} \sim 50$ TeV. The corresponding S_{LIV} for pair production will be 0.51 for 100 TeV and 0.01 for 1000 TeV (instead of corresponding values 10^{-2} and 10^{-5} given in Table 1).

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